# **Calculation of space charge density in negative corona based on finite element iteration and sound pulse method**

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**Space charge distribution is a valuable indicator of ion flow field and is widely utilized to assess electromagnetic environment for transmission line. FEM is widely used to solve this problem. However, Kaptzov assumption in FEM assumes that onset voltage is unchangeable after corona occurs, this is not tally with the actual situation. This paper adopted the finite element iteration method (FEIM), which uses empirical formula instead of assumptions, to calculate the charge density. The surface field intensity is not constant in the empirical formula because the empirical takes the effects of space charge on surface field into consideration. This is more consistent with actual situation. In order to verify the accuracy of the algorithm, sound pulse method is applied to measure the charge density around the conductor in laboratory. Simulation results shows that the charge density obtained in this study agree with the data in the experiment, which verifies the validity of the empirical formula for calculation of space charge density.**

*Index Terms***—empirical formula, finite element iteration algorithm, sound pulse method, space charge density.**

# I. INTRODUCTION

WITH the development of the UHVDC transmission<br>technology, the problem of corona loss and the technology, the problem of corona loss and the electromagnetic environment interference is taken into consideration more and more seriously. These problems are closely associated with corona discharge, and the space charge density is the core of corona discharge. So the research of space charge density around conductor line is of great significance.

Sarma [1] proposed the flux line method to solve the ion flow field around UHVDC transmission line. But the flux line method depends on the Deutsch hypothesis, which means that space charge only affects the field intensity, but not affects the direction of electric field. This hypothesis does not accord with actual situation seriously, and causes the error of the calculation results. Janischewskyj [2] introduced the finite element method for the analysis of dc ion flow field. This method was mainly based on Kaptzov hypothesis and abandoned the Deutsch hypothesis, and this made some improvement on the accuracy of the result. But Kaptzov hypothesis refers to the electric field on the surface of conductor line keeps unchangeable after corona discharge happens, and it's a critical boundary condition when calculating the ion flow. This is still a gap with actual situation and also reduces the accuracy of results. So how to adopt criteria which is more accord with actual situation to replace assumption is still a puzzle.

This paper adopted the finite element iteration method [3] to analyze the distribution of space charge density. Instead of using Deutsch and Kaptzov assumption, this method uses an empirical formula [4] to indicate the surface field intensity. The surface field in the empirical formula is influenced by space charge and not a constant. Compared with the hypothesis, empirical formula is better to conform to the actual situation. At the same time, the experiment of measuring the space charge density based on sound pulse

method [5] in laboratory is used to verify the accuracy of method adopted in this paper.

## II.MATHEMATICAL MODEL

Without considering the effects of wind, the governing equation of ionized field is as shown below:

$$
\nabla^2 \varphi = -\frac{\rho}{\varepsilon_0} \tag{1}
$$

$$
\nabla \cdot \mathbf{j} = 0 \tag{2}
$$

$$
\mathbf{j} = k \rho \mathbf{E} \tag{3}
$$

$$
E = -\nabla \varphi \tag{4}
$$

where  $\varphi$  represents the potential,  $\rho$  is the space charge density,  $\varepsilon_0$  is the vacuum dielectric constant, *j* is the electric current density, *k* is ion mobility, and *E* is the field intensity.

 Because of the complexity of the corona discharge, some simplified conditions are adopted. (1) The ply of the corona layer can be neglected, (2) the ion mobility is unchangeable, (3) the surface field intensity meets the empirical formula.

$$
E_c = E_0[1.1339 - 0.1678(U/U_0) + 0.03(U/U_0)^2] (5)
$$

With the assumptions above, Equations (1) to (4) can be expressed as

$$
\nabla \cdot (\nabla \varphi \nabla^2 \varphi) = 0 \tag{6}
$$

In view of the governing equation, charge density and the electric field intensity have a coupling effect. Therefore, the controling equation can be decomposed into the equations:

$$
-\nabla \bullet (\nabla \varphi) = \rho / \varepsilon_0 \tag{7}
$$

$$
\nabla \bullet (\rho \nabla \varphi) = 0 \tag{8}
$$

Equations (7) and (8) are the form of the Poisson equation, so it can be calculated by the finite element method.

The initial charge density has a significant influence on calculation accuracy. Therefore, we adopted the following equation to calculate the initial value.

$$
\rho(x, y) = \frac{\rho_0}{[1 + (x^2 + (y - h)^2)/h^2]^2}
$$
(9)

where *h* is the distance between the ground and wire, *x* and *y* are the coordinates of each node, and  $\rho_0$  is unchangeable.

If the convergence criterion is met in the process of calculation, the algorithm ended.

$$
\delta \rho < 5\% \tag{10}
$$

If Equation (10) is not met, the charge density at each node can be updated by the equation

$$
\rho_n = \rho_p + \delta \rho \tag{11}
$$

$$
\delta \rho = \alpha \frac{Es}{Ec} \frac{-\delta \varphi}{\frac{1}{2} (\varphi_A + \varphi_B)} \rho_p \tag{12}
$$

where  $\rho_n$  is the new charge density,  $\rho_p$  is the charge density in the previous step, *E*<sup>c</sup> is the electric field intensity calculated by (5), *E*cal is the average value of Equations (6) and (7), and *α* is 1.02. The flowchart of calculation process is shown in Fig.1.



Fig. 1. Flowchart of the finite iteration algorithm

## III. VERIFICATION OF ALGORITHM

In order to verify the validity of the algorithm, a platform of space charge measurement has been set up in the laboratory, as shown in Fig. 2. This platform is based on sound pulse method. The concrete experiment principle is that trigger signal controls the ultrasonic transducer to produce sound wave to vibrate the space charge around conductor line. The vibration of electric charge will produce electric field signal. This field signal can be received by electric field antenna. After that, according to the relation between charge density and electric field intensity, we induce a decoupling algorithm to calculate the distribution of the space charge density.



Fig. 2. The structure of experimental platform.

 The background noise is processed by superimposed averaging method and adaptive filtering method. After that, the decoupling algorithm is used to calculate the space charge density around the testing conductor line.



 From the results, the experimental result and simulation result have good consistency. They both have an attenuation trend form conductor line to ground connection. Around the conductor line, the experiment and simulation of the maximum charge density is  $1.51 \times 10^{-4}$  C/m<sup>3</sup> and  $1.31 \times 10^{-4}$  $C/m<sup>3</sup>$ , respectively. Close to the ground, the maximum charge density is  $0.20 \times 10^{-4}$  C/m<sup>3</sup> and  $0.13 \times 10^{-4}$  C/m<sup>3</sup>, respectively. From the value of the charge density, there is still a deviation. In the process of experiment, the presence of background noise leads to inaccurate measurement. Noise makes the distortion of electric field signal, thereby it affects the calculation of charge density. This reason makes the deviation existing in the final result. But from the trend and order of magnitudes of the result, the simulation method is applicable to calculate the space charge density.

#### IV. CONCLUSION

In this work, the FEIM uses empirical formula instead of assumption to calculate the charge density. It is found that the charge density decrease form the conductor to ground, which has same trends and order of magnitudes with the sound pulse method. This shows that the empirical formula applied in algorithm is feasible to calculate the charge density.

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